D.R.G.Government Degree College

Tadepalligudem, West Godavari District

V SEMESTER

PHYSICS PAPER- V(B)

MODERN PHYSICS

STUDY MATERIAL

(ENGLISH MEDIUM)

Prepared by

K.V.Ganesh Kumar

Assistant Professor of Physics

E-Mail: ganesh.kurakula@gmail.com

www.ganeshkurakula.blogspot.com

AdiKavi Nanayya University::Rajamahendravaram

III B.Sc. : Physics Paper V(B)- Semester-V

MODERN PHYSICS

No. of Credits: 03
3 Hour/Week
Total Hours : 45

UNIT-I (9 hrs)

1. Atomic and molecular physics

Introduction –Drawbacks of Bohr's atomic model. Vector atom model and Stern-Gerlach experiment - quantum numbers associated with it. L-S and j- j coupling schemes. Zeeman effect(Definition only) -Raman effect, hypothesis, Stokes and Anti Stokes lines. Quantum theory of Raman effect. Experimental arrangement –Applications of Raman effect.

UNIT-II (9 hrs)

2. Matter waves & Uncertainty Principle

Matter waves, de Broglie's hypothesis - wavelength of matter waves, Properties of matter waves - Davisson and Germer experiment – Heisenberg's uncertainty principle for position and momentum (x and p) & Energy and time (E and t).

UNIT-III (9 hrs)

3. Quantum (wave) mechanics

Basic postulates of quantum mechanics-Schrodinger time independent and time dependent wave equations-derivations. Physical interpretation of wave function. Eigen functions, Eigen values. Application of Schrodinger wave equation to particle in one dimensional infinite box.

UNIT-IV(9 hrs)

4. General Properties of Nuclei

Basic ideas of nucleus -size, mass, charge density (matter energy), binding energy, magnetic moment, electric moments. Liquid drop model and Shell model (qualitative aspects only) - Magic numbers.

5. Radioactivity decay

Alpha decay: basics of a-decay processes. Theory of a-decay, Gamow's theory, Geiger Nuttal law.β-decay, Energy kinematics for β-decay, positron emission, electron capture, neutrino hypothesis.

UNIT-V (9 hrs)

6. Crystal Structure

Amorphous and crystalline materials, unit cell, Miller indices, reciprocal lattice, types of lattices, diffraction of X-rays by crystals, Bragg's law, experimental techniques, Laue's method.

7. Superconductivity

Introduction - experimental facts, critical temperature - critical field - Meissner effect – Isotope effect - Type I and type II superconductors - applications of superconductors.

Unit-I

Atomic & Molecular Physics

Vector Atom Model

Vector atom model is proposed to explain the drawbacks of Bohr and Somerfield atomic models. Two important postulates of vector atom model are

1. Space quantization
2. Electron Spin

1. Space quantization:

According to vector atom model, motion of the electron around the nucleus is three dimensional. Hence a third quantum number is required to describe the state of the electron in addition to the two quantum numbers described by the Somerfield atomic model. The third quantum number determines the orientation of the orbit in three dimensional space. According to classical physics, the orientation of the orbit of the electron is random. But according to quantum mechanics, the orientation of the orbit cannot have all possible values but posses only certain allowed values. Hence the orbits are quantized both in magnitude and direction. This is known as space quantization.

2. Electron spin:

Uhlenback and Goudsmit proposed that in addition to the orbital motion, Electron has spin motion also to explain the anomalous Zeeman effect and the doublet fine structure of alkali spectra. This is known as spinning electron. Hence the electron has spin angular momentum in addition to the orbital angular momentum. According to quantum theory, the spin angular momentum is also quantized like orbital angular momentum.

According to vector atom model, the physical quantities which describe the state of the electron Orbital angular momentum, spin angular momentum and the corresponding magnetic moments are quantized vector quantities. Hence this model is known as vector atom model.

Quantum numbers associated with vector atom model

1. Principal quantum number (n):

- This quantum number defines the principal orbit of the electron.
- This is also known as total quantum number (n).
- Values of principal quantum number are integers starting from 1.

$$n = 1,2,3 \ldots.$$

- 'n' values represent the shells of the electron. $n = 1$ indicates K shell, $n = 2$ indicates L shell, $n = 3$ indicates M shell…
- Maximum number of electrons in a shell is $2n^2$.

2. Orbital quantum number (l):

- This quantum number defines the shape of the orbit and orbital angular momentum.
- Hence this quantum number is also known as orbital angular momentum quantum number (l).
- Values of 'l' are integers from 0 to $(n-1)$

$$l = 0 \quad to \quad (n-1)$$

- l values represent the sub shells or orbitals of the electrons. $l = 0$ represents s orbital, . $l = 1$ represents p orbital, $l = 2$ represents d orbital.

- Orbital quantum number defines the quantization of orbital angular momentum.

$$p_l = \sqrt{l(l+1)}\frac{h}{2\pi}$$

3. **Spin quantum number (s):**
   - This quantum number defines the spin angular momentum of the electron.
   - Hence this quantum number is also known as Spin angular momentum quantum number ($\boldsymbol{s}$).
   - Spin quantum number s has only two values $s = +\frac{1}{2}$ and $s = -\frac{1}{2}$
   - Spin quantum number defines the quantization of spin angular momentum.

$$p_s = \sqrt{s(s+1)}\frac{h}{2\pi}$$

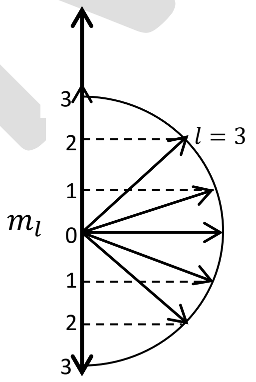
4. **Total angular quantum number (j):**
   - The sum of orbital angular momentum and spin angular momentum is known as total angular momentum.
   - This quantum number defines the total angular momentum of the electron.
   - Hence this quantum number is total angular quantum number (j).
   - Values of j are half integral values ranging from $(l-s)$ to $(l+s)$

$$j = (l-s) \;\; to \;\; (l+s)$$

5. **Magnetic orbital quantum number ($\boldsymbol{m_l}$):**
   - This quantum number defines the quantization of orbital angular momentum in the presence of magnetic field.
   - According to quantum mechanics, the projections of orbital angular momentum in the direction of magnetic field are not continuous and can have only integral values from $-l$ to $+l$.
   - This condition defines the directions of orbital angular momentum. These projections are called magnetic orbital quantum numbers (m_l).

$$m_l = -l, -l+1, -l+2, \dots 0, 1, 2, \dots l-2, l-1, 1$$

   - Number of allowed m_l is $(2l+1)$.



6. **Magnetic spin quantum number ($\boldsymbol{m_s}$):**
   - This quantum number defines the quantization of spin angular momentum in the presence of magnetic field.
   - According to quantum theory, the projections of spin angular momentum are not continuous and can have only two values $m_s = -\frac{1}{2}, m_s = +\frac{1}{2}$.
   - These projections are called magnetic spin quantum numbers (m_s).

7. Total magnetic quantum number (m_j):

- This quantum number defines the quantization of total angular momentum quantum number in the presence of magnetic field.
- According to quantum theory, the projections of total angular momentum quantum number in the direction of magnetic field are not continuous and can only have half integral values from $-j$ to $+j$. These projections are called total magnetic quantum numbers (m_j).

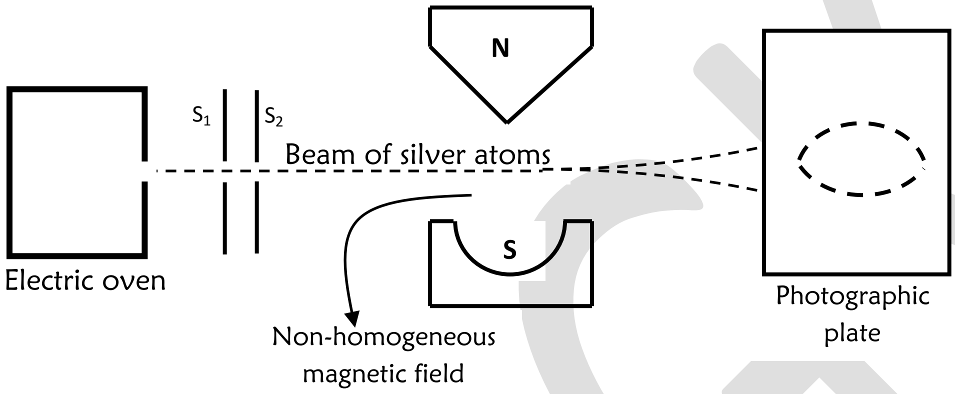
$$m_j = -j, -j+1, -j+2, \ldots 0, 1, 2, \ldots j-2, j-1, 1$$

- Number of allowed m_j values is $(2j+1)$.

Stern-Gerlach Experiment

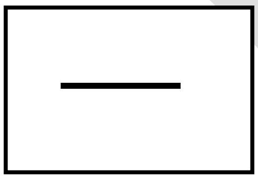
Aim: The aim of Stern-Gerlach experiment is to verify the concepts space quantization and electron spin proposed by vector atom model.

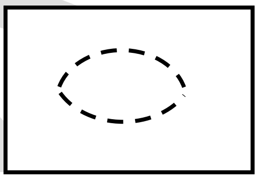
Experimenal arrangement:

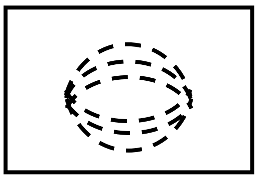


The experimental arrangement of Stern-Gerlach experiment is shown in figure. Silver atoms emitted from the electric oven pass through the slits S_1, S_2 to produce a beam of silver atoms. The beam of silver atoms is passed through a non-homogeneous magnetic field. The shape of the magnetic poles is such that the magnetic field intensity increases from the centre towards the upper magnetic pole and hence a non-homogeneous magnetic field is obtained. The beam of silver atoms is deflected by the magnetic field and falls on the photographic plat. The whole arrangement is enclosed in a evacuated glass chamber.

Result obtained:


In the absence of magnetic field


In the presence of non-homogeneous magnetic field


In the presence of homogeneous magnetic field

In the absence of magnetic field, the beam of silver atoms forms a narrow strip as shown in figure. But in presence of non-homogeneous magnetic field, the strip splits up into two components. The splitting of silver atomic beam into two components proves the space quantization postulate proposed by vector atom model.

Proving the postulates of vector atom model from the result obtained

Since the atomic number of silver is 47, it has an unpaired electron in its outermost shell $5\,s$. This unpaired electron is responsible for the magnetic moment μ_J of silver atom. When these silver atoms having a magnetic moment μ_J pass through the non-homogeneous magnetic field, unequal and opposite forces act on them. Hence the silver atoms undergo transverse displacement. If the directions of magnetic moments are continuous as per classical mechanics, then the silver atoms form a broad continuous band as shown in figure. But in the Stern-Gerlach experiment, only two narrow strips are obtained. Hence it is clear that the directions of magnetic moments are not continuous but they are quantized according to quantum mechanics and have only two allowed directions

In Silver atoms, the magnetic moment due to orbital motion is zero. But it is clear from the transverse displacement of the silver atoms that their magnetic moment is not zero. This magnetic moment of silver atoms must have been produced by the spin motion of the electrons. Hence electrons have spin motion in addition to the orbital motion. This verifies the concept of electron spin proposed by vector atom model.

Expression for transverse displacement of silver atoms is given by

$$D = \frac{1}{2}\frac{\mu_J}{m}\left(\frac{dB}{dx}\right)\left(\frac{L}{v}\right)^2$$

In the above equation $\mu_J = \mu \cos\theta$

If the direction of magnetic moment is not quantized and has continuous values from, the transverse displacement would also be continuous and a broad continuous spectrum would have been observed. But in Stern-Gerlach experiment, only two narrow strips are obtained. This verifies the concept of space quantization.

L-S Coupling and j-j Coupling

Every electron in the atom has Orbital angular momentum and spin angular momentum. In multi-electron atoms, the total angular momentum depends on the interaction between the Orbital angular momentum and Spin angular momentum.

Hence there are two methods to calculate the total angular momentum.

1. L-S Coupling
2. J-j Coupling

1. L-S Coupling:

This type of coupling takes place more often and hence it is known as Normal Coupling. In this Coupling, orbital angular moments of different electrons combine to produce a resultant orbital angular momentum vector L. Similarly Spin angular moments of different electrons combine to produce a resultant spin singular momentum vector S. These two vectors L and S combine to produce the total angular momentum J.

$$\mathrm{L} = \mathrm{l}_1 + \mathrm{l}_2 + \mathrm{l}_3 + \cdots \ldots$$
$$\mathrm{S} = \mathrm{s}_1 + \mathrm{s}_2 + \mathrm{s}_3 + \cdots \ldots.$$
$$\mathrm{J} = \mathrm{L} + \mathrm{S}$$

When $\mathrm{L} > \mathrm{S}$ Number of possbile values of J is $2\mathrm{S} + 1$

When $\mathrm{L} < \mathrm{S}$ Number of possible values of J is $2\mathrm{L} + 1$

This is known as multiplicity.

- ✓ $\mathrm{L} = 0$ indicates S State
- ✓ $\mathrm{L} = 1$ indicates P State
- ✓ $\mathrm{L} = 2$ indicates D State
- ✓ $\mathrm{L} = 3$ indicates F state

If $\mathrm{S} = 0$ then, Multiplicity $2\mathrm{S} + 1 = 1$. Hence J has only one value. This is known a Singlet

If $\mathrm{S} = \frac{1}{2}$ then, Multiplicity $2\mathrm{S} + 1 = 2$. Hence J has two values. This is known as a doublet.

If $\mathrm{S} = 1$ then, Multiplicity $2\mathrm{S} + 1 = 3$. Hence J has three values. This is known as a triplet.

Example:

If $\mathrm{L} = 1, \mathrm{S} = \frac{1}{2}$

Since $\mathrm{L} > \mathrm{S}$ Multiplicity $2\mathrm{S} + 1 = 2$

$\mathrm{L} - \mathrm{S} = 1 - \frac{1}{2} = \frac{1}{2}$ $\quad$ $\mathrm{L} + \mathrm{S} = 1 + \frac{1}{2} = \frac{3}{2}$

Hence the values of J are $\frac{1}{2}$ or $\frac{3}{2}$

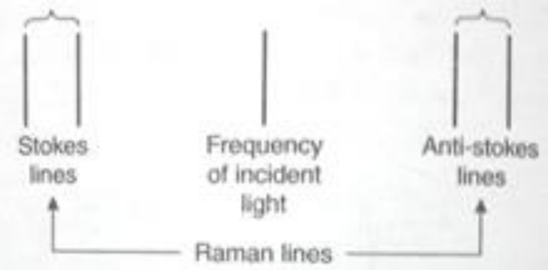
The spectral terms are $2_{\mathrm{P}_{1/2}}$ and $2_{\mathrm{P}_{3/2}}$

2. j-j Coupling: This type of coupling occurs when the spin-orbit interaction is strong. In this coupling, orbital and spin angular moments of individual electrons combine to produce $j_1, j_2, j_3, \ldots..$ Then these total angular momentum vectors combine to produce to the resultant total angular momentum vector J.

$$j_1 = l_1 + s_1, j_2 = l_2 + s_2, j_3 = l_3 + s_3$$
$$J = j_1 + j_2 + j_3 + \cdots ....$$

Raman Effect

When light is incident on a transparent medium, the scattered light consists of frequencies smaller and greater than the frequency of incident light. This phenomenon is known as Raman Effect and the spectrum is known as Raman Spectrum.

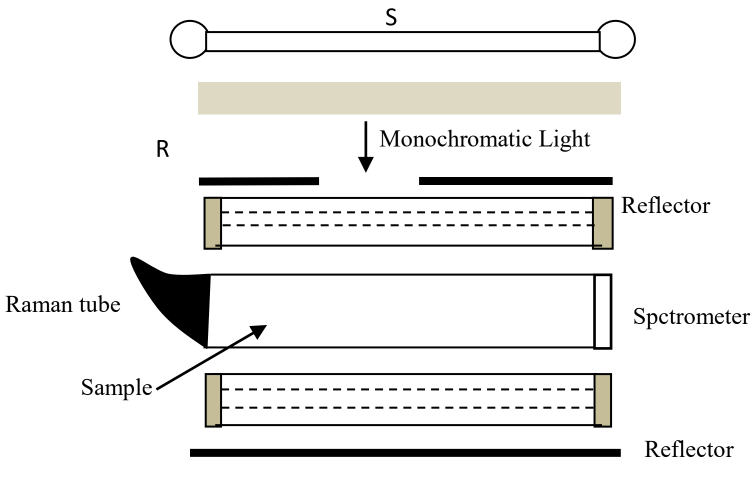


- ✓ Lines having smaller frequency are called Stokes lines and the lines having larger frequency are called anti-stokes lines.
- ✓ Raman shift is independent of the frequency of the incident light and depends only on the nature of the material. Hence the properties of materials can be studies using Raman Effect.

Experimental arrangement to study Raman Effect

Experimental arrangement for observation of Raman Effect is shown in figure. It mainly consists of three parts.

1. Source of Light
2. Raman tube
3. Spectrograph



- ✓ S is a Mercury vapor lamp.
- ✓ Light emitted by this lamp passes through a filter F to become monochromatic
- ✓ The monochromatic light enters the Raman tube after passing through a metallic reflector R.
- ✓ The metallic reflector increases the amount of light entering the Raman tube.
- ✓ Raman tube is in the shape of a cylinder with diameter of 1-2 cm and length 10-15 cm.
- ✓ The scattered light comes out through the plane end of the glass tube. It is observed using a spectrograph.
- ✓ Other end of the glass tube is horn shaped and coated with black color to avoid multiple reflections.
- ✓ Cold water is circulated in the jacket surrounding the Raman tube to avoid overheating.
- ✓ The liquid sample is placed in Raman tube. If the sample is in solid state, it is dissolved in paraffin liquid.

Unit-I

Matter Waves

De Broglie postulated that matter also exhibits dual nature like radiation since nature loves symmetry. Hence matter exhibits wave nature also in addition to particle nature.

According to de Broglie's postulate, every moving particle behaves like a wave. This wave is called de Broglie wave or matter wave.

Wavelength of de Broglie wave associated with a particle of mass 'm' moving with velocity 'v' is given by

$$\lambda = \frac{h}{mv}$$

Wavelength of de Broglie wave or de Broglie wavelength

From Max Planck's Quantum theory, energy of a photon is given by

$$E = h\upsilon = \frac{hc}{\lambda} \longrightarrow ①$$

Here c is the speed of light and λ is the wavelength.

From Einstein's mass-energy relation

$$E = mc^2 \longrightarrow ②$$

From equations 1 and 2

$$mc^2 = \frac{hc}{\lambda}$$

$$\lambda = \frac{h}{mc}$$ Here mc is the momentum of photon.

Momentum of a particle of mass m moving with velocity v

$$p = mv$$

Hence de Broglie wavelength

$$\boldsymbol{\lambda = \frac{h}{mv} = \frac{h}{p}}$$

1.If E is the kinetic energy of the particle, then

$$E = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

$$p = \sqrt{2mE}$$

$$\lambda = \frac{h}{\sqrt{2mE}}$$

2.When a particle of charge q is accelerated through a potential difference of V.

$$\lambda = \frac{h}{\sqrt{2mqV}}$$

3.If the particle is in thermal equilibrium at a temperature T, then $E = \frac{3}{2}kT$

$$\lambda = \frac{h}{\sqrt{3mkT}}$$

Properties of matter waves

1.Lighter is the particle, longer is the wavelength of de Broglie wave associated with it.

2.Smaller is the velocity of the particle, longer is wavelength of de Broglie wave associated with it.

3.Matter waves are generated by the motion of particles. Stationary particles do not exhibit wave nature.

4.velocity of matter waves is greater than the velocity of light.

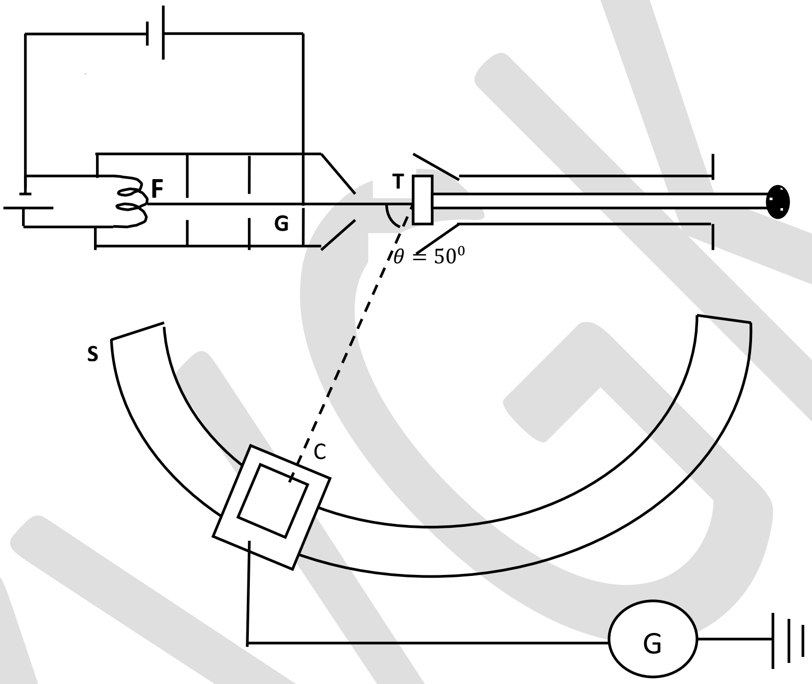
5.Matter waves are independent of the charge of the particle.

6.Wave nature of matter leads to Heisenberg's uncertainty principle.

7.A particle do not exhibit wave nature and particle nature simultaneously.

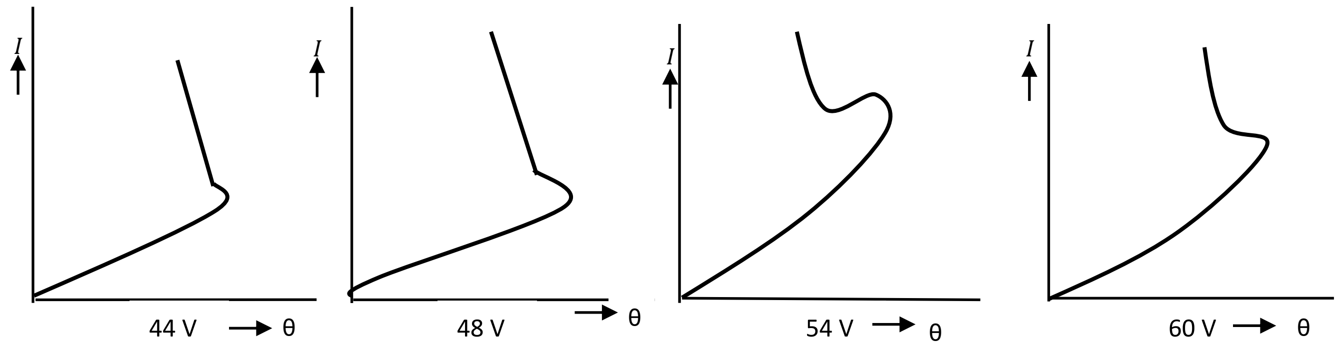
Davisson-Germer Electron diffraction experiment

Aim: According to de Broglie's postulate, every moving particle behaves like a wave. This wave is called de Broglie wave or matter wave.



Aim of this experiment is to prove the existence of matter waves.

- G is an electron gun as shown in figure.
- The electron gun G contains a tungsten filament F from which electrons are emitted due to thermo ionic emission.
- These electrons are accelerated by applying suitable voltage.
- These electrons are passed through the slits S_1, S2 to obtain a narrow beam of electrons.
- The beam of electrons falls on the nickel crystal N and gets diffracted in different directions.
- The number of electrons diffracted in each direction is obtained using a Faraday's cylinder C. A galvanometer G is connected to the Faraday's cylinder.
- Faraday's cylinder is moved to different positions on the scale S and galvanometer current is measured in each direction at constant voltage. The galvanometer current is directly proportional to the number of electrons diffracted in that direction.



Graphs are drawn by taking the angle of diffraction θ on X-axis galvanometer current on Y-axis at different voltages. A bump appears in the curve at 44 volts as shown in the graph. The bump increases with increasing voltage and becomes prominent at 54 volts. The bump disappears above this voltage.

The bump in the curve at 54 volts is due to the diffraction of electrons. This proves the existence of matter waves.

The de Broglie wavelength of electrons at voltage V is given by $\lambda = \frac{12.26}{\sqrt{V}} A^0$

$$\lambda = \frac{12.26}{\sqrt{54}} = 1.67\ A^0 \longrightarrow ①$$

For plane diffraction grating

$$n\lambda = d \sin\theta$$

For Nickel crystal $= 2.15\ A^0$, $n = 1$ Hence $\lambda = 2.15\ \sin 50^0 = 1.65\ A^0 \longrightarrow$ ②

The values of wavelengths obtained from equations 1 and 2 are nearly equal. Hence it is clear that electrons are also behaving like waves. Hence de Broglie's postulate of matter waves is true.

Unit-III

Quantum Mechanics

Basic postulates of Quantum Mechanics

According to de-Broglie's postulate, every moving particle behaves like a wave. Hence this wave nature of particles must be considered to explain the physical properties of any quantum system. Hence this branch of Physics is known as Wave Mechanics or Quantum Mechanics.

- ❖ Every physical system has a wave function. This wave function completely describes any state of the system. This wave function is denoted by ψ.

Properties of wave function:

- ✓ Wave function must be continuous at all points.
- ✓ Wave function must be finite and single valued at all points.
- ✓ First derivatives of the wave function must be continuous and single valued.

- ❖ Every observable quantity of the system can be expresses by an operator

  If $\widehat{\boldsymbol{G}}$ represents the operator of a physical quantity, then the expectation value of the physical quantity is given by

$$< G >= \int_{-\infty}^{\infty} \psi^* \boldsymbol{G}\, \psi \; dV$$

Physical significance of wave function

Let ψ be the wave function of a particle. Let ψ^* be the complex conjugate of the wave function . According to Born's explanation, the value of $\psi^*\psi = |\psi|^2$ gives the position probability density of the particle at a given point.

Position probability density of the particle= $\psi^*\psi = |\psi|^2$

If the value of $|\psi|^2$ is large, then the probability of finding the particle at that point is also large. If the value of $|\psi|^2$ is small, then the probability of finding the particle at that point is also small.

The total probability of finding the particle somewhere is unity.

$$\int_{-\infty}^{\infty} |\psi|^2 \; dV = 1$$

This above equation is knwon as Normalization equation.

Eigen values and Eigen functions

Let ψ be the wave function of a system. Let $\widehat{\boldsymbol{G}}$ be the operator corresponding to the physical quantity G. The following condition must be satisfied for the physical quantity to be quantized

$$\widehat{\boldsymbol{G}}\, \psi = \lambda\, \psi$$

Here λ is a real number. This is the experimentally determined value of the physical quantity. Hence λ is known as Eigen value. Wave function ψ satisfying this condition is known as Eigen function.

Since a system has number of wave functions, the above equation can be expressed as

$$\widehat{\boldsymbol{G}}\, \psi_n = \lambda_{\,n} \psi_n$$

Hence the experimentally determined value of the physical quantity is equal to one of the values $\lambda_{\,n}$.

Time independent Schrodinger wave equation

According to de-Broglie's postulate, every moving particle behaves like a wave. This is known as matter wave or de-Broglie wave. Wavelength of de-Broglie wave associated with a particle of mass m moving velocity v is given by

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

- Wave equation which explains the wave nature of particles is known as Schrodinger wave equation.

Let ψ be the wave function of particle with position coordinates (x, y, z)at any time t.

General wave equation is given by

$$\frac{\partial^2\psi}{\partial t^2} = v^2\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)$$

$$\nabla^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

$$\boldsymbol{\frac{\partial^2\psi}{\partial t^2} = v^2\nabla^2\psi} \longrightarrow \boxed{1}$$

Let the solution of the above equation be

$$\psi = \psi_0 \sin \omega t$$

$$\psi = \psi_0 \sin 2\pi\nu t$$

$$\frac{\partial\psi}{\partial t} = \psi_0(2\pi\nu)\cos 2\pi\nu t$$

$$\frac{\partial^2\psi}{\partial t^2} = -\psi_0(2\pi\nu)^2 \sin 2\pi\nu t = -4\pi^2\nu^2(\psi_0 \sin 2\pi\nu t) = -4\pi^2\nu^2\psi$$

$$\frac{\partial^2\psi}{\partial t^2} = -4\pi^2\nu^2\psi \longrightarrow \boxed{2}$$

From equations 1 and 2 $\qquad v^2\nabla^2\psi = -4\pi^2\nu^2\psi$

$$\nabla^2\psi = -\frac{4\pi^2\nu^2}{v^2}\psi$$

$$\nabla^2\psi + \frac{4\pi^2\nu^2}{v^2}\psi = 0$$

$$\nu = \frac{v}{\lambda}$$

$$\frac{\nu}{v} = \frac{1}{\lambda}$$

$$\nabla^2\psi + \frac{4\pi^2}{\lambda^2}\psi = 0$$

Wavelength of matter waves $\qquad \lambda = \frac{h}{mv}$

$$\nabla^2\psi + \frac{4\pi^2}{\left(\frac{h}{mv}\right)^2}\psi = 0$$

$$\nabla^2\psi + \frac{4\pi^2m^2v^2}{h^2}\psi = 0$$

Let be V the potential energy and E be the total energy of the particle.

$$\frac{1}{2}mv^2 = E - V$$

$$mv^2 = 2(E - V)$$

$$m^2v^2 = 2m(E - V)$$

$$\nabla^2\psi + \frac{4\pi^2.2m(E - V)}{h^2}\psi = 0$$

$$\nabla^2\psi + \frac{8\pi^2m(E - V)}{h^2}\psi = 0$$

$$\nabla^2\psi + \frac{2m(E - V)}{\left(\frac{h^2}{4\pi^2}\right)}\psi = 0$$

$$\boxed{\boldsymbol{\nabla^2\psi + \frac{2m(E - V)}{\hbar^2}\psi = 0}}$$

This is known as time independent Schrodinger wave equation.

$$\nabla^2\psi = -\frac{2m(E-V)}{\hbar^2}\psi$$

$$-\frac{\hbar^2}{2m}\nabla^2\psi = (E-V)\,\psi$$

$$-\frac{\hbar^2}{2m}\nabla^2\psi = E\psi - V\psi$$

$$-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = E\psi$$

$$\left(-\frac{\hbar^2}{2m}\nabla^2 + V\right)\psi = E\psi$$

$$\boldsymbol{\hat{H}\psi = E\psi}$$

$$\hat{H} = \text{Hamiltonian Opearator} = -\frac{\hbar^2}{2m}\nabla^2 + V$$

Time dependent Schrodinger wave equation

According to de-Broglie's postulate, every moving particle behaves like a wave. This is known as matter wave or de-Broglie wave. Wavelength of de-Broglie wave associated with a particle of mass m moving velocity v is given by

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

- Wave equation which explains the wave nature of particles is known as Schrodinger wave equation.

Let ψ be the wave function of particle with position coordinates (x, y, z)at any time t.

General wave equation is given by

$$\frac{\partial^2\psi}{\partial t^2} = v^2\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)$$

$$\nabla^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

$$\boldsymbol{\frac{\partial^2\psi}{\partial t^2} = v^2\nabla^2\psi} \quad \boxed{1}$$

Let the solution of the above equation be

$$\psi(x, y, z, t) = \psi_0(x, y, z)e^{-i\omega t}$$

$$\psi = \psi_0 e^{-i\omega t}$$

$$\frac{\partial\psi}{\partial t} = \psi_0(-i\omega)e^{-i\omega t}$$

$$\frac{\partial\psi}{\partial t} = -i\omega\,\psi_0 e^{-i\omega t}$$

$$\frac{\partial\psi}{\partial t} = -i\,2\pi\nu\,\psi$$

According to Quantum theory, $E = h\nu$

$$\nu = \frac{E}{h}$$

$$\frac{\partial\psi}{\partial t} = -i\,2\pi\frac{E}{h}\,\psi = -i\,\frac{E}{\hbar}\psi$$

$$\frac{\partial\psi}{\partial t} = -i\,\frac{E}{\hbar}\psi$$

$$\hbar\frac{\partial\psi}{\partial t} = -i\,E\psi$$

$$i\hbar\frac{\partial\psi}{\partial t} = E\psi$$

$$\boldsymbol{E\psi = i\hbar\frac{\partial\psi}{\partial t}}$$

Time independent Schrodinger wave equation

$$\left(-\frac{\hbar^2}{2m}\nabla^2 + V\right)\psi = E\psi$$

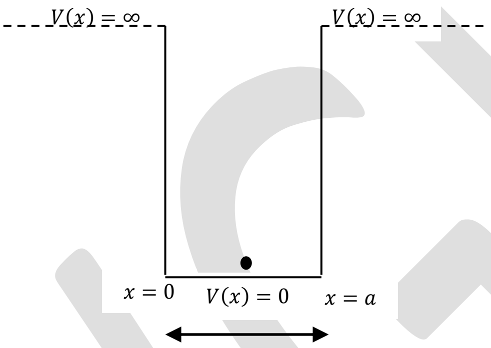
$$\left(-\frac{\hbar^2}{2m}\nabla^2 + V\right)\psi = i\hbar\frac{\partial\psi}{\partial t}$$

$$\boxed{\boldsymbol{\hat{H}\psi = i\hbar\frac{\partial\psi}{\partial t}}}$$

$$\hat{H} = \text{Hamiltonian Opeartor} = -\frac{\hbar^2}{2m}\nabla^2 + V$$

This is known as Schrodinger time independent wave equation.

Particle in one dimensional potential box



Consider a particle of mass m moving in a one dimensional potential barrier as shown in figure.

Potential is infinity at the walls of the barrier and is zero between them.

$$V(x) = 0, \quad 0 < x < a$$

$$V(x) = 0, \quad x = 0, x = \infty$$

Time independent Schrodinger wave equation,

$$\frac{d^2\psi}{dx^2} + \frac{2m(E-V)}{\hbar^2}\psi = 0$$

Since the potential is zero inside the barrier, $V(x) = 0$

$$\frac{d^2\psi}{dx^2} + \frac{2mE}{\hbar^2}\psi = 0$$

$$Let\ \frac{2mE}{\hbar^2} = k^2$$

$$\frac{d^2\psi}{dx^2} + k^2\psi = 0$$

Solution of the above equation,.

$$\psi = A\sin kx + B\cos kx \qquad \boxed{1}$$

According to boundary conditions,

$$\psi(x) = 0 \quad when \quad x = 0 \qquad \boxed{2}$$

$$\psi(x) = 0 \quad when \quad x = a \qquad \boxed{3}$$

From equations 1 and 2

$$0 = A(0) + B$$

$$B = 0 \qquad \boxed{4}$$

$$\therefore\ \psi = A\sin kx$$

From equations 3 and 4

$$0 = A\sin ka$$

$$\sin ka = 0$$

$$ka = 0, \pi, 2\pi, 3\pi$$

$$ka = n\pi, \qquad n = 1,2,3$$

$$k = \frac{n\pi}{a}$$

$$\therefore \psi(x) = A\ \sin\frac{n\pi x}{a}$$

From Normalization condition

$$\int_0^a |\psi(x)|^2 dx = 1$$

$$A^2\int_0^a \sin^2\frac{n\pi x}{a}\ dx = 1$$

$$\frac{A^2}{2}\int_0^a \left(1-\cos\frac{2n\pi x}{a}\right) dx = 1$$

$$\frac{A^2}{2}\left[\int_0^a 1\, dx - \int_0^a \cos\frac{2n\pi x}{a}\ dx\right] = 1$$

$$\frac{A^2}{2}(a) = 1$$

$$A^2 = \frac{2}{a}$$

$$A = \sqrt{\frac{2}{a}}$$

$$\psi(x) = A\sin\frac{n\pi x}{a}$$

$$\boldsymbol{\psi(x) = \sqrt{\frac{2}{a}}\sin\frac{n\pi x}{a}, \quad n = 1,2,3 \ldots\ldots}$$

The above equation represents the wave function of a particle moving in a one dimensional potential barrier.

Eigen values:

$$k^2 = \frac{2mE}{\hbar^2}$$

$$k = \frac{n\pi}{a}$$

$$\left(\frac{n\pi}{a}\right)^2 = \frac{2mE}{\hbar^2}$$

$$\frac{n^2\pi^2}{a^2} = \frac{2mE}{\hbar^2}$$

$$E = \frac{n^2\pi^2\hbar^2}{2ma^2}$$

$$\boldsymbol{E_n = \frac{n^2\pi^2\hbar^2}{2ma^2}, \qquad n = 1,2,3 \ldots\ldots}$$

Hence energy of the particle in a one dimensional potential box is quantized.

$$\boldsymbol{E_1 = \frac{\pi^2\hbar^2}{2ma^2}}$$

$$E_2 = \frac{4\pi^2\hbar^2}{2ma^2} = 4E_1$$

$$E_3 = \frac{9\pi^2\hbar^2}{2ma^2} = 9E_1$$

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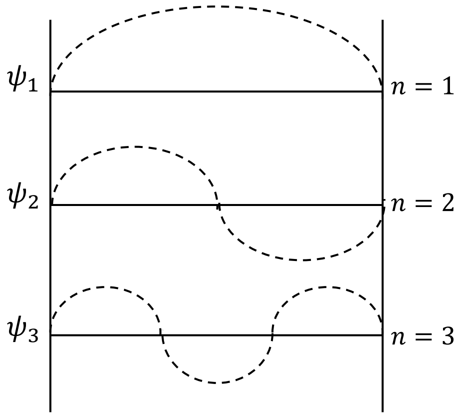
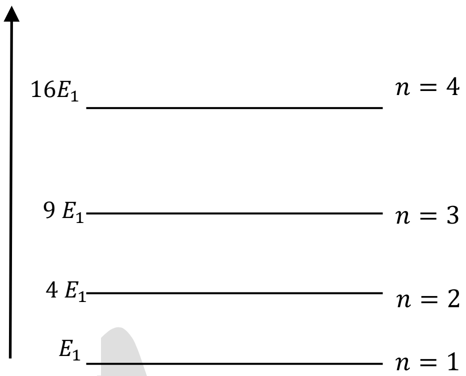


Eigen functions



Eigen values

Unit-IV
Nuclear Physics

Basic Properties of Nucleus

Magnetic Dipole Moment (μ):

Magnetic dipole moment of an electron due to its spin is given by

$$\mu_e = \frac{e\,\hbar}{2m_e}$$

This is called Bohr Magneton.

Similarly the magnetic dipole moment of Proton due to its spin is

$$\mu_N = \frac{e\,\hbar}{2m_p}$$

Here μ_N is called the Nuclear Magneton.

Since m_p=1836m_e, the value of Nuclear Magneton is 1/1836 of Bohr Magneton.

The magnetic dipole moment of Neutron should be zero since it is neutral. But experimentally it is found that neutron has a magnetic dipole moment given by

$\mu_n = -1.9128\,\mu_N$.

This is because Proton and Neutron are not two different particles but they are two different states of a single particle called Nucleon.

Electric Quadrapole Moment (Q):

Shape of the atomic nucleus is not an exact sphere. It is an ellipsoid of revolution.

The Electric Quadrapole moment of a nucleus is a measure of the deviation of the shape of the nucleus from a sphere.

The electric Quadrapole moment of a nucleus with atomic number Z is given by

$$Q = \frac{2}{5}\,Z\,e\,(\,b^2 - a^2)$$

i. For a perfect sphere a=b, hence Q=0
ii. For an Oblate spheroid, a>b, hence Q is negative
iii. For a Prolate Spheroid, a<b, hence Q is Positive.

It is found experimentally that the value of Q is zero for nuclei with spin I=0 or ½

Nuclear Spin (I):

Proton and neutron have an intrinsic spin angular momentum of like electron. Since the nucleons revolve around the centre of the nucleus, they posses orbital angular momentum also. The vector sum of these two angular momenta is the total angular momentum of the nucleus. This total angular momentum of the nucleus is called the nuclear spin.

Binding energy of a nucleus

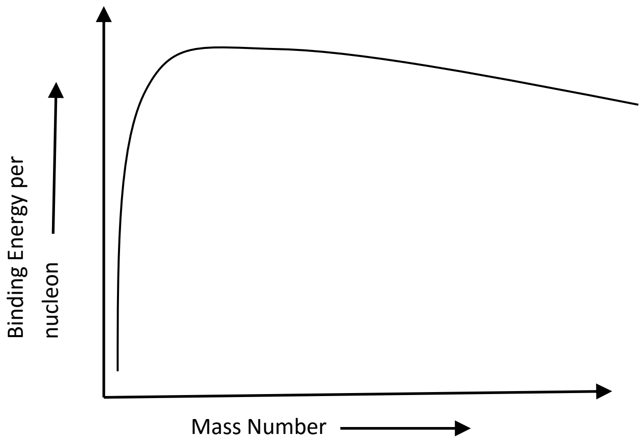
It is found that the actual mass of a nucleus is always less than the total of the protons and neutrons in it. This difference in mass is known as Mass defect. This mass is converted into energy as per Einstein's Mass-Energy equivalence. This energy is responsible for holding protons and neutrons together in the nucleus and hence known as Binding energy. If the binding energy of a nucleus is more, then the nucleus is highly stable.

The Binding energy of a nucleus with atomic number Z and neutron number N is given by

$$B.E = \left[\left(Z\,m_p + N\,m_n\right) - M\right]C^2$$

Here M is the actual mass of the nucleus.

A graph is drawn between mass number (A) on X-axis and the binding energy per nucleon on Y-axis as shown in figure.



It is clear from the graph that the binding energy increases rapidly with mass number A and reaches a maximum value of 8.8 Mev at A=56. The binding energy then decreases slowly with increasing mass number and reaches a value of 8.4 Mev at A=140. It drops to a value of 7.6 Mev above A=140. Hence it is clear that nuclei with mass numbers between 50 and 140 are extremely stable.

Binding Energy of Deuteron

The heavy isotope of Hydrogen containing one Proton and Neutron is called Deuteron.

Mass of Proton= 1.007825 amu

Mass of Neutron= 1.008665 amu

Total Mass= 1.007825+1.008665= 2.016490 amu

Actual Mass of Deuteron=2.014104

Mass Defect = 2.016490-2.014104=0.002386 amu

Binding Energy = 0.002386 x 931 = 2.23 Mev

Hence 2.23Mev of energy is released when a Proton and a Neutron combine to form a Deuteron nucleus.

Properties of Nuclear forces

An atomic nucleus contains positively charged Protons and neutrons. There will be Coulomb's repulsive forces between the Protons which tend to make the nucleus unstable. Hence there must be strong attractive forces between the nucleons for the nucleus to remain stable. These strong attractive forces are called Nuclear Forces. The properties of nuclear forces are

1. Nuclear forces are short range forces. Nuclear force acts only if the separation between the two nucleons is less than 10^{-15}m.
2. Nuclear forces are independent of charge. The nuclear force between two protons or between two neutrons or between one proton and one neutron is the same
3. Nuclear forces depend on the relative spin orientations of the nucleons. Nuclear force between two nucleons with parallel spins is stronger than that between two nucleons with anti-parallel spins.
4. Nuclear forces are saturated forces. Hence each nucleon in the nucleus interacts with only a certain number of nucleons nearest to it.
5. Nuclear forces are the strongest forces in nature.
6. Nuclear forces are non-central forces as they depend on spin of nucleons.

Nuclear Models

The exact nature of nuclear forces acting between the nucleons is not known completely. Hence to explain the important properties of the nucleus, nuclear models were proposed. Two important nuclear models are

- Liquid drop model
- Shell Model

Liquid Drop Model

According to Liquid drop model, the atomic nucleus is equivalent to an electrically charged liquid drop. The idea of liquid drop model is based on the following facts.

- The nuclear density is the same for all the nuclei.
- The binding energy per nucleon is almost constant for all the nuclei.

According to liquid drop model the similarities between the nucleus and the liquid drop are as follows.

1. Both the nucleus and the atomic nucleus are spherical in shape.
2. The nucleons in the nucleus behave like molecules in the liquid drop.
3. The density of nucleus is independent of its volume like the liquid drop.
4. The nucleons interact only with its nearest neighbours like the molecules in a liquid drop.
5. The potential barrier on the surface of the nucleus is similar to the surface tension forces in the liquid drop.
6. The nucleons deep inside the nucleus are attracted from all sides. But the nucleons on the surface of the nucleus are attracted from only side. Hence the binding energy of the nucleus on the surface of the nucleus is less.

Merits:

1. Liquid drop model is successful in explaining the nuclear reactions and nuclear fission.
2. The atomic masses and binding energies of the nucleus can be accurately calculated using this model.

Demerits:

Liquid drop model failed to explain the existence of magic numbers.

Shell Model

It was observed that nuclei containing proton number or neutron number equal to 2,8,20,28,50,82 and 126 are extremely stable. These numbers are called Magic numbers. The shell model of the nucleus is developed from the concept of magic numbers.

According to shell model of the nucleus, the nucleons in the nucleus occupy certain discrete energy levels or shells like the electrons in an atom. The nucleons are filed in the energy levels basing on the Pauli's exclusion principle. According to this principle two nucleons with opposite spins are placed in the same shell.

The shell model is based on the following two assumptions.

1. Each nucleon in the nucleus moves in a common force field of all the remaining nuclei.
2. The energy levels are filled according to Pauli's principle.

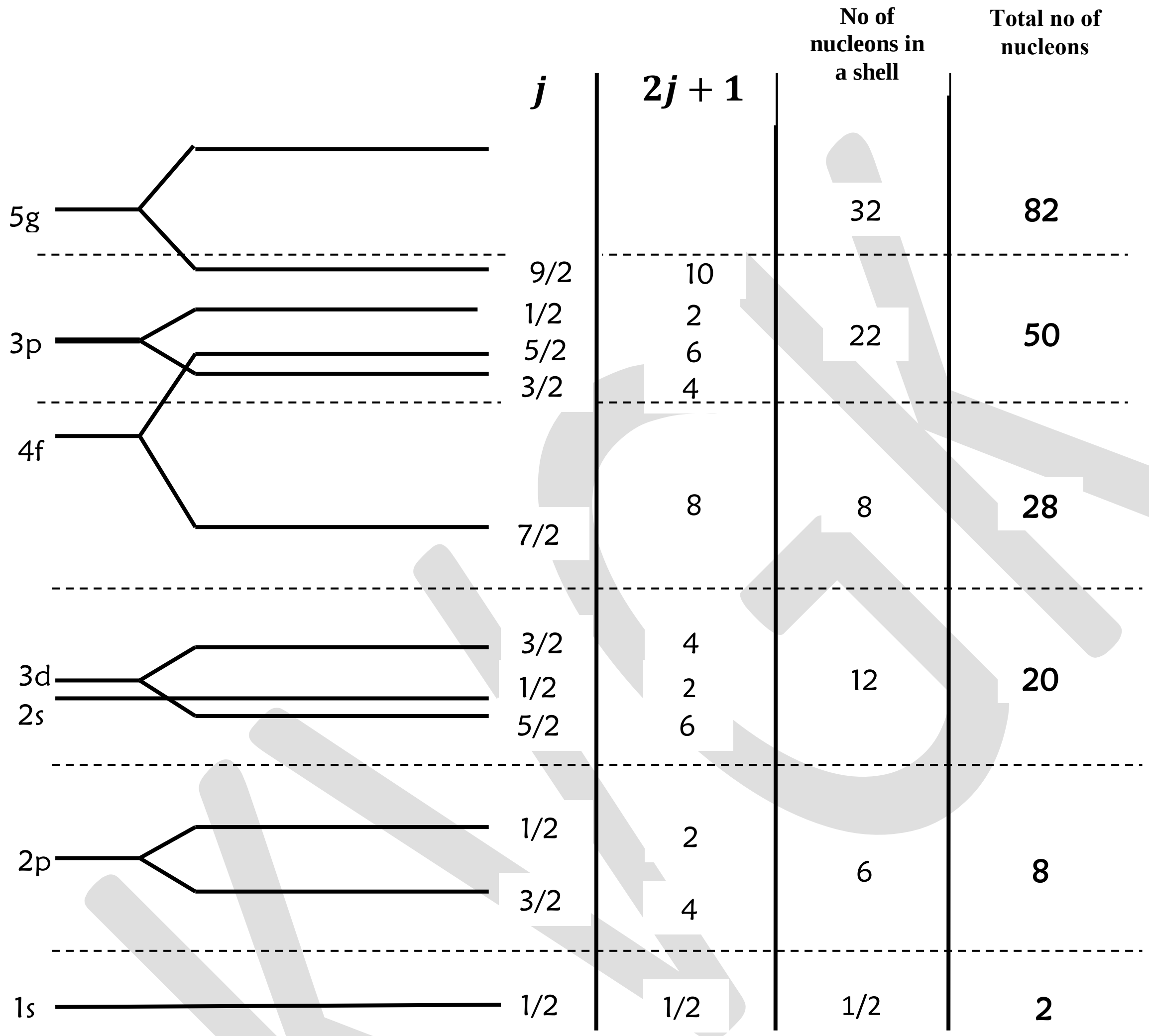
The Schrödinger's wave equation for a nucleon of mass M is given by

$$\frac{d^2}{dr^2}(Rr)+\frac{2M}{h^2}\left(E-V-\frac{l(l+1)h^2}{2Mr^2}\right)(Rr)=0$$

The allowed discrete energy levels of the nucleons are obtained by solving the Schrödinger's wave equation for the nuclear potential well. The vector sum of the Orbital angular momentum l and the spin angular momentum s is called the total angular momentum j.

$$j = l + s \quad 0r \quad j = l - s$$

Each j value contains (2j+1) energy levels. The energy levels of the nucleons are shown in figure.



It is clear that nuclei containing proton number or neutron number equal to the magic numbers 2,8,20,28,50,82 and 126 form closed shells. Hence they are extremely stable.

<u>Merits</u>:

1. Successfully explained the magic numbers,
2. Explained the angular momenta, magnetic moments and quadrapole moments of nuclei.

<u>Demerits</u>:

1. Could not explain the large quadrapole moments of some nuclei.
2. Could not explain the spherical shapes of many nuclei.

Gamow theory of alpha decay (or) Geiger-Nuttal law

If a nucleus decays by the emission of alpha particle, it is known as alpha decay. Generally alpha decay takes place in nuclei with high atomic number. In such nuclei, the repulsion force between the protons is very high. Hence the nucleus decays by the emission of alpha particle.

$$^{238}_{92}U \longrightarrow \quad ^{234}_{90}Th + \ ^{4}_{2}He$$

Gamow theory of alpha decay:

The energy required for the alpha particle to escape from the nucleus is 27 MeV. But experimentally, it is found that the energy of alpha particle emitted from natural radioactive substances is less than 4 MeV. Hence according to classical physics, alpha particle cannot escape from the nucleus. But Gamow proved that according to quantum mechanics, there is a finite probability for the alpha particle to escape from the nucleus even if its energy is less than the potential barrier. This is known as Gamow's theory.

Two important postulates of Gamow theory are

1. Alpha particle is preformed inside the nucleus.
2. Alpha particle is in constant motion and collides frequently with the walls of the barrier. In each collision, there is a finite probability for the alpha particle to escape from the nucleus.

If ν is the collision frequency, λ is the decay constant and P is the probability

$$\lambda = \nu\, P$$

$$\lambda = \left(\frac{v}{2r_0}\right) P$$

$$\log_e \lambda = \log_e \left(\frac{v}{2r_0}\right) + \log_e P$$

According to WKB approximation

$$\log_e P = -\frac{2}{\hbar}\int_{r_0}^{r_1} \sqrt{2m[V(r) - E]}\, dr$$

$$V(r) = \frac{2Ze^2}{4\pi\epsilon_0 r}$$

$$\log_e P = -\frac{2}{\hbar}\int_{r_0}^{r_1} \sqrt{2m\left[\frac{2Ze^2}{4\pi\epsilon_0 r} - E\right]}\, dr$$

At $r = r_1$, $E = \frac{2Ze^2}{4\pi\epsilon_0 r_1}$

$$\frac{2Ze^2}{4\pi\epsilon_0} = Er_1$$

$$\log_e P = -\frac{2}{\hbar}\int_{r_0}^{r_1} \sqrt{2m\left[\frac{Er_1}{r} - E\right]}\, dr = -\frac{2}{\hbar}\sqrt{2mE}\int_{r_0}^{r_1} \sqrt{\left[\frac{r_1}{r} - 1\right]}\, dr$$

$$\log_e P = -\frac{2}{\hbar}\sqrt{2mE}\int_{r_0}^{r_1} \sqrt{\left[\frac{r_1}{r} - 1\right]}\, dr$$

$$\int_{r_0}^{r_1} \sqrt{\left[\frac{r_1}{r} - 1\right]}\, dr = r_1\left[\cos^{-1}\sqrt{\frac{r_0}{r_1}} - \sqrt{1 - \frac{r_0}{r_1}}\sqrt{\frac{r_0}{r_1}}\right]$$

$$\cos^{-1}\sqrt{\frac{r_0}{r_1}} = \frac{\pi}{2} - \sqrt{\frac{r_0}{r_1}}$$

$$\sqrt{1-\frac{r_0}{r_1}} = 1$$

$$\therefore \int_{r_0}^{r_1}\sqrt{\left[\frac{r_1}{r}-1\right]}\,dr = r_1\left[\frac{\pi}{2} - \sqrt{\frac{r_0}{r_1}} - \sqrt{\frac{r_0}{r_1}}\right] = r_1\left[\frac{\pi}{2} - 2\sqrt{\frac{r_0}{r_1}}\right]$$

$$E = \frac{2Ze^2}{4\pi\epsilon_0 r_1}$$

$$r_1 = \frac{2Ze^2}{4\pi\epsilon_0 E}$$

$$\int_{r_0}^{r_1}\sqrt{\left[\frac{r_1}{r}-1\right]}\,dr = \left(\frac{2Ze^2}{4\pi\epsilon_0 E}\right)\left[\frac{\pi}{2} - 2\sqrt{\frac{4\pi\epsilon_0 E r_0}{2Ze^2}}\right]$$

$$\log_e P = -\frac{2}{\hbar}\sqrt{2mE}\int_{r_0}^{r_1}\sqrt{\left[\frac{r_1}{r}-1\right]}\,dr = \left(\frac{2Ze^2}{4\pi\epsilon_0 E}\right)\left[\frac{\pi}{2} - 2\sqrt{\frac{4\pi\epsilon_0 E r_0}{2Ze^2}}\right]$$

$$\log_e P = \frac{4e}{\hbar}\left(\frac{m}{\pi\epsilon_0}\right)^{1/2} Z^{1/2} r_0^{1/2} - \frac{e^2}{\hbar\epsilon_0}\left(\frac{m}{2}\right)^{1/2} ZE^{-\frac{1}{2}} = 2.97\,Z^{1/2} r_0^{1/2} - 3.95\,ZE^{-\frac{1}{2}}$$

$$\log_e \lambda = \log_e\left(\frac{v}{2r_0}\right) + \log_e P$$

$$\boxed{\log_e \lambda = \log_e\left(\frac{v}{2r_0}\right) + 2.97\,Z^{1/2} r_0^{1/2} - 3.95\,ZE^{-\frac{1}{2}}}$$

This is known as Geiger-Nuttal law. From the above equation it is clear that, the decay constant varies exponentially with the energy of alpha particle.

β – Decay

If a nucleus spontaneously emits a beta particle (electron or positron) to obtain stability, it is known as β – decay.

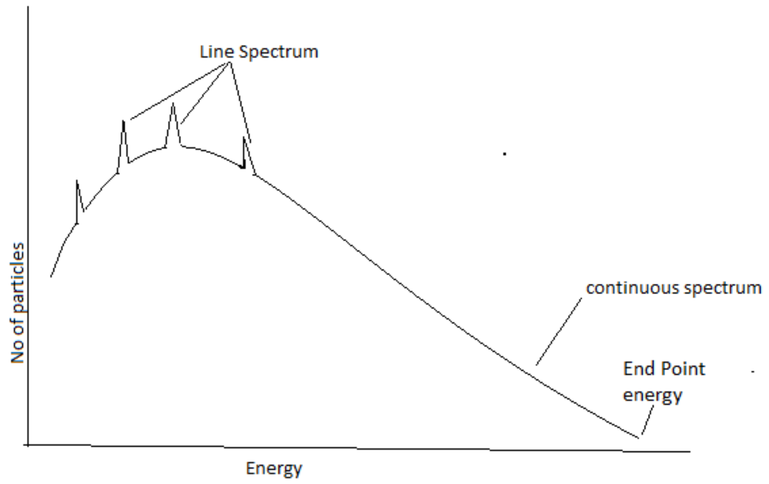
β – Decay tells us that neutron and proton are not two different particles but they are two different states of a single particle called nucleon. β – Decay occurs when a neutron is converted to a proton or vice-versa.

$$n \longrightarrow p + e^- + \bar{v} \qquad\qquad p \longrightarrow n + e^+ + \nu$$

β – Ray Spectrum

A graph drawn between the no of electrons emitted and their energy in β – Decay is shown in figure. The graph is a continuous curve with some sharp lines at certain energy values. Hence it is clear that β- particles emitted from a radioactive substance have continuous energy values. Hence this is called continuous spectrum. But some substances also give line spectrum in addition to the continuous spectrum as shown in figure. Hence the β- Ray spectrum contains two parts.

1. Continuous Spectrum
2. Line spectrum or Discrete Spectrum



1. Continuous Spectrum:

In continuous spectrum, the energy of emitted β- particles varies from zero to a certain maximum value. The maximum energy is known as end point energy. This end point energy depends on the nature of the emitter. The continuous spectrum is formed by the electrons emitted by the nucleus during β-decay. These electrons are called primary electrons. Hence the continuous spectrum depends on the actual decay.

2. Line spectrum:

We know that Gamma rays are emitted from the nucleus when a nucleon in the nucleus jumps from one energy level to the other. The electrons in the K, L, M and N shells of the atom absorb these gamma rays to produce secondary electrons with discrete energy values. This process is called internal conversion. The sharp lines observed in the β-ray spectrum are due to the process of internal conversion. Hence the line spectrum in the β-ray spectrum is due to the secondary electrons.

Difficulties with β-ray Spectrum

1. We know that nucleons are arranged in discrete energy levels in the nucleus. So the β-particles emitted from the nucleus during β-decay must have discrete energy values. But the β-particles are emitted with continuous energy values. The β-ray spectrum is continuous instead of being discrete.
2. The law of conservation of angular momentum is not obeyed in β-decay.

   The spin of β-particle is ½. So the spin of the nucleus must change by ½ when a beta particle is emitted from it. But the spin of the nucleus either remained unchanged or changed by an integral value instead of ½.
3. The law of conservation of linear momentum is also not obeyed in β-decay.

Pauli's Neutrino hypothesis of β-decay

Pauli proposed Neutrino hypothesis to explain the difficulties with β-ray spectra. According to this hypothesis, in the process β-decay, a neutral particle called Neutrino is also emitted along with the β-particle. The mass of Neutrino is negligible and its spin is ½.

We know that nucleus contains positively charged protons and uncharged neutrons. β-particles are emitted from the nucleus when a proton is converted into neutron or vice-versa. When a neutron is converted into a proton, electron and neutrino are created.

Neutron $\longrightarrow$ Proton + Electron + Neutrino

$$0^{n^1} \longrightarrow 1^{H^1} + -1^{e^0} + \nu$$

Mass 1 $\longrightarrow$ 1 + 0 + 0

Charge 0 $\longrightarrow$ +1 + -1 + 0

The difficulties with β-ray spectra can be explained with this theory.

1. In the process of β-decay, the β-particle and neutrino are emitted form the nucleus. The total energy of these two particles is constant and is equal to the end point energy. The total energy is shared between the proton and the neutrino. If the energy of the neutrino is zero, then the β-particle is emitted with the end point energy. The amount of energy shared by the neutrino is not constant and changes continuously. Hence the energy of the emitted β-particles also changes continuously. So the spectrum is continuous.
2. In the process of β-decay, the mass number remains constant. Hence the angular momentum of the nucleus is not changed. But the angular momentum of the emitted β-particle is $\frac{1}{2}\left(\frac{h}{2\pi}\right)$. Hence neutrino must have an angular momentum of $-\frac{1}{2}\left(\frac{h}{2\pi}\right)$ for the angular momentum of the nucleus to remain constant.

Unit-V

Crystal Structure

Solids are of two types. Crystalline and amorphous.

Crystals: if the atoms in a solid are arranged in a regular and periodic manner, then the solid is known as a crystal. In crystals the atoms have long range order.

Ex: Gold, Silver, Aluminium, Diamond, Quartz etc

Amorphous Solids: If the atoms in a solid are arranged in an irregular fashion, then the solid is known as an amorphous solid. In amorphous substances, atoms have short range order.

Ex: Glass, Rubber, Plastic etc.

S.No	Crystalline Solids	Amorphous Solids
1	If the atoms in a solid are arranged in a regular and periodic manner, then the solid is known as a crystal.	If the atoms in a solid are arranged in an irregular fashion, then the solid is known as an amorphous solid
2	In crystals the atoms have long range order.	In amorphous substances, atoms have short range order.
3	Ex: Gold, Silver, Aluminium, Diamond, Quartz etc	Ex: Glass, Rubber, Plastic etc..
4	Crystals have different physical properties in different directions. Hence crystals are anisotropic.	Physical properties of amorphous solids are the same in all directions. Hence they are isotropic.
5	Crystals have a sharp melting point.	Amorphous solids do not have sharp melting point.
6	The cooling curves of crystals are not continuous. They have breaks.	The cooling curves of amorphous solids are smooth without any breaks.

Unit Cell:

It is not necessary to describe the positions of all the atoms in the crystal to explain its structure. It will be sufficient to describe the positions of a certain group of atoms to explain the crystal structure. This group of atoms is called a unit cell.

Unit cell of a crystal is defined as the smallest volume element which produces the entire structure of the crystal by translational repetition in three directions.

Unit cell of a crystal is not unique. A crystal can more than one unit cell. If the number of atoms per unit cell is one, then the unit cell is called a primitive cell.

Crystal Lattice:

The geometry of the crystal can easily studied by replacing the atoms with lattice points. This hypothetical arrangement of lattice points to describe the geometry of the crystal is known as crystal lattice. This is a geometrical concept only.

Every point in the crystal lattice has the same environment. The entire crystal structure is obtained if an atom or group of atoms are attached to every point in the crystal lattice. This group of atoms is called the Base.

Crystal structure= Crystal lattice + Basis.

Miller Indices

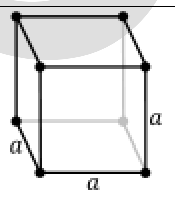
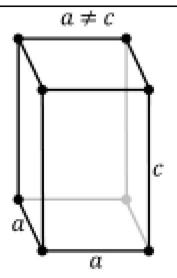
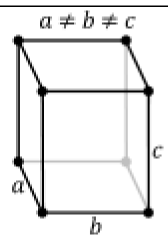
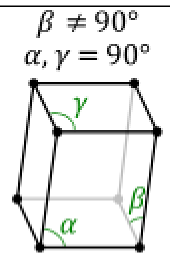
A crystal is made up of a large number of parallel lattice planes. The Orientation of these lattice planes is given by a set of integers known as Miller Indices. Miller indices of a lattice planes are represented by (h,k,l).

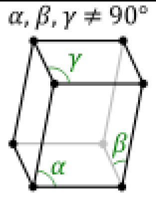
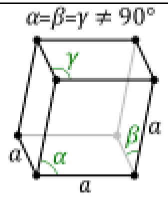
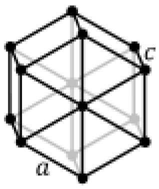
The Miller indices of a lattice plane are defined as the smallest ratio of the reciprocals of the intercepts made by the plane with the axes.

Rules for finding the Miller Indices:

1. The intercepts made by the lattice plane with the three axes p,q,r are determined.
2. Reciprocals of these intercepts 1/p, 1/q, 1/r are determined.
3. These reciprocals are multiplied by the least common multiple of p,q,r to obtain h,k,l
4. The set of integers (h,k,l) denotes the Miller indices of the lattice plane.

Crystals are classified into 7 types basing on their geometrical arrangement. These are called crystal systems.

S.No	Crystal System	Lattice Parameters	Unit Cell	Examples
1	Simple Cube	a=b=c, $\alpha = \beta = \gamma = 90°$		Nacl, CaF_2, Diamond, ZnS
2	Tetragonal	$a = b \neq c$, $\alpha = \beta = \gamma = 90°$		$NiSO_4$, SnC_2
3	Ortho Rhombic	$a \neq b \neq c$, $\alpha = \beta = \gamma = 90°$		KNO_3,$BaSO_4$, $SnSO_4$
4	Monoclinic	$a \neq b \neq c$, $\alpha = \beta = 90° \neq \gamma$		Na_2SO_4, $FeSO_4$,Borax

5	Triclinic	$a \neq b \neq c$, $\alpha \neq \beta \neq \gamma \neq 90°$		$CuSO_4$, $K_2Cr_2O_7$
6	Trigonal	$a = b = c$, $\alpha = \beta = \gamma \neq 90°$		$CaSO_4$, Quartz, Calcite
7	Hexagonal	$a = b \neq c$, $\alpha = \beta = 90; \gamma = 120°$		SiO_2, AgI

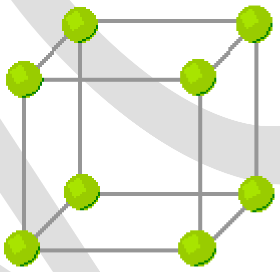
Bravais Lattices

Bravais showed that there are 14 ways of arranging points in a space such that the environment looks the same from each point. These 14 space lattices are called Bravais Lattices.These 14 Bravais lattices belong to 7 crystal systems.There are three Bravais lattices for a cubic system.They are

1. **Simple Cubic:**

   In Simple cubic structure, the atoms are arranged only at the eight corners of the cube.So the total number of lattice points in simple cubic structure is 8.

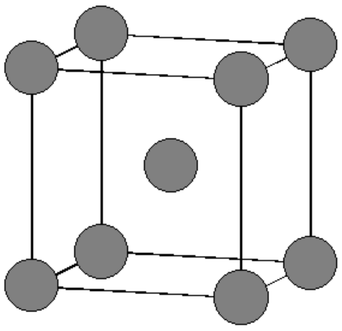
   Hence the number of atoms per unit cell is 8 x 1/8 = 1.



2. **Body Centred Cubic (B.C.C):**

   In Body Centred Cubic structure, one atom is located at the centre of the cube in addition to the eight atoms at the corners. So the total number of lattice points in Body centred cubic structure is 9.

   The number of lattice points per unit cell is $8 \times 1/8 + 1 \times 1 = 2$



3. **Face Centred Cubic (F.C.C):**

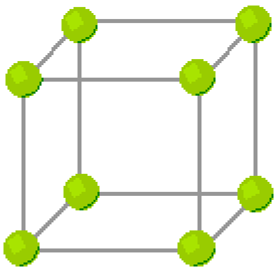
   In Face centred cubic structure, there is one atom at the centre of each of the six faces of the cube in addition to the eight atoms at the corners. So the total no of lattice points in face centred cubic structure is 14.

The number of lattice points per unit cell= 8 x 1/8 + 6 x 1/2 = 4.

Simple cubic Structure:

In Simple cubic structure, the atoms are arranged only at the eight corners of the cube.So the total number of lattice points in simple cubic structure is 8.

Hence the number of atoms per unit cell is 8 x 1/8 = 1.



1. Let ‘r’ be the atomic radius and ‘a’ be the lattice constant.

   In simple cubic structure the atoms touch each other along the edges.

   Hence 2r=a

   r= a/2
2. The ratio of the volume occupied by atoms in a unit cell to the total volume of the unit cell is known as atomic packing fraction.

   Packing fraction= volume occupied by the atoms / volume of the unit cell

$$= \frac{1\times\frac{4}{3}\pi r^3}{a^3}$$

$$= \frac{4\pi r^3}{3a^3} = \frac{4\pi\, a^3}{24a^3} = \frac{\pi}{6} = 0.52 = 52\%$$

3. The number of equidistant nearest neighbours of an atom in a unit cell is known as the coordination number.

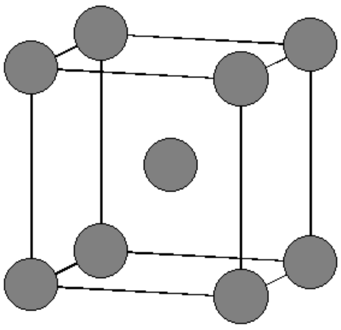
In Simple cubic structure, the atoms are arranged only at the eight corners of the cube.Consider an atom at the origin (0,0,0). It has six nearest neighbours. Their coordinates are (a,0,0) ,(-a,0,0),(0,a,0), (0,-a,0), (0,0,a), (0,0,-a).

Hence the coordination number is 6.

Body Centred Cubic (B.C.C):

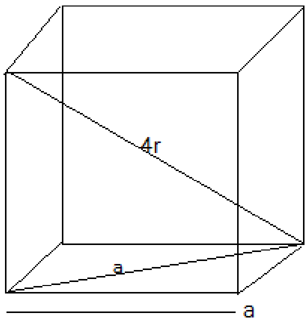
In Body Centred Cubic structure, one atom is located at the centre of the cube in addition to the eight atoms at the corners. So the total number of lattice points in Body centred cubic structure is 9.

The number of lattice points per unit cell is 8 x 1/8 + 1 x 1 = 2



1. Let ‘r’ be the atomic radius and ‘a’ be the lattice constant.

   In simple cubic structure the atoms touch each other along the body diagonal.



$$AG^2 = AD^2 + DG^2$$

$$a^2 + a^2 = 2a^2$$

$$(4r)^2 = a^2 + 2a^2 = 3a^2$$

$$r = \frac{\sqrt{3}\,a}{4} \quad \text{or} \quad a = \frac{4r}{\sqrt{3}}$$

2. The ratio of the volume occupied by atoms in a unit cell to the total volume of the unit cell is known as atomic packing fraction.

   Packing fraction= volume occupied by the atoms / volume of the unit cell

$$= \frac{2\times\frac{4}{3}\pi\,(\frac{\sqrt{3}}{4}a)^3}{a^3}$$

$$= \frac{\sqrt{3}\,\pi}{8} = 0.68 = 68\ \%$$

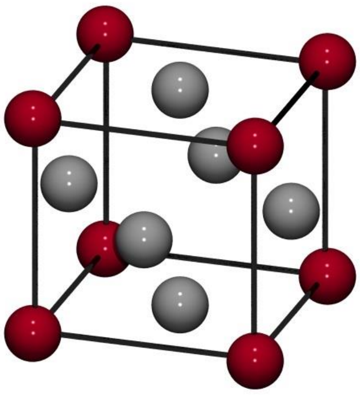
3. The number of equidistant nearest neighbours of an atom in a unit cell is known as the coordination number.

   In Body Centred Cubic structure, one atom is located at the centre of the cube in addition to the eight atoms at the corners. It has eight nearest neighbours.

   Hence the coordination number is 8.

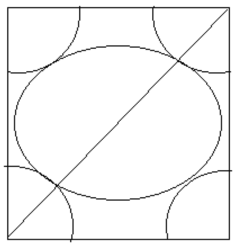
Face Centred Cubic (F.C.C):

In Face centred cubic structure, there is one atom at the centre of each of the six faces of the cube in addition to the eight atoms at the corners. So the total no of lattice points in face centred cubic structure is 14.



The number of lattice points per unit cell= 8 x 1/8 + 6 x 1/2 = 4.

1. Let ‘r’ be the atomic radius and ‘a’ be the lattice constant.In simple cubic structure the atoms touch each other along the face diagonal.



$$DB^2 = DC^2 + CB^2$$

$$(4r)^2 = a^2 + a^2 = 2a^2$$

$$r = \frac{\sqrt{2}\,a}{4}$$

$$a = \frac{4r}{\sqrt{2}}$$

2. The ratio of the volume occupied by atoms in a unit cell to the total volume of the unit cell is known as atomic packing fraction.

   Packing fraction= volume occupied by the atoms / volume of the unit cell

$$= \frac{4\times\frac{4}{3}\pi\,(\frac{\sqrt{2}a}{4})^3}{a^3}$$

$$= 4\times\,\frac{\pi\,a^3\sqrt{2}}{24\,a^3}$$

$$= \frac{\pi\,\sqrt{2}}{6}$$

$$= 0.74 = 74\ \%$$

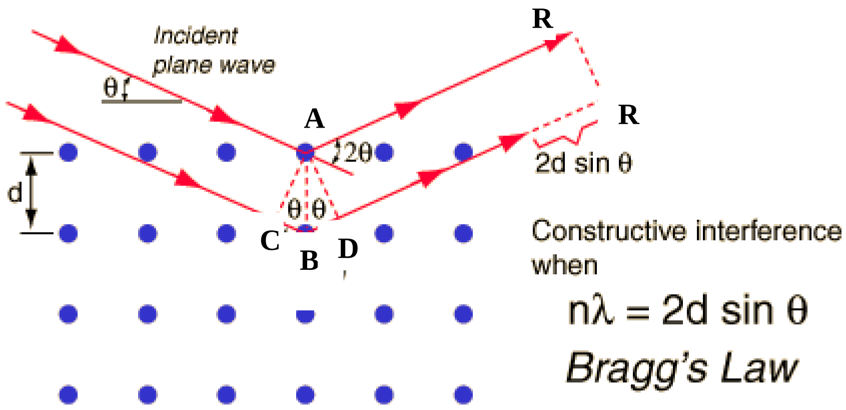
3. The number of equidistant nearest neighbours of an atom in a unit cell is known as the coordination number. Consider an atom at the origin (0,0,0). It has twelve nearest neighbours. Hence the coordination number is 12.

X-ray Diffraction

X-rays cannot be diffracted using a normal grating since their wavelength is very small. But Laue suggested that X-rays can be diffracted using a crystal. We know that the atoms in a crystal are arranged in a regular and periodic manner. Hence the crystal serves as a natural three dimensional grating.

Bragg's Law

We know that a crystal lattice contains a number of parallel lattice planes. These parallel planes are called Bragg planes. Bragg's law explains the diffraction of X-rays by a crystal.



Let a parallel beam of X-rays of wavelength λ is incident on the Bragg planes with a glancing angle θ as shown in figure. Let 'd' be the inter planar spacing. X-rays are reflected from point 'A' in the direction AR from the upper plane and from point 'B' in the direction BR from the lower plane. There will be a path difference between the two reflected rays.

$$\text{path difference} = CB+BD$$

$$\text{From figure } CB=BD=d\sin\theta$$

$$\text{Total Path difference} = 2d\sin\theta$$

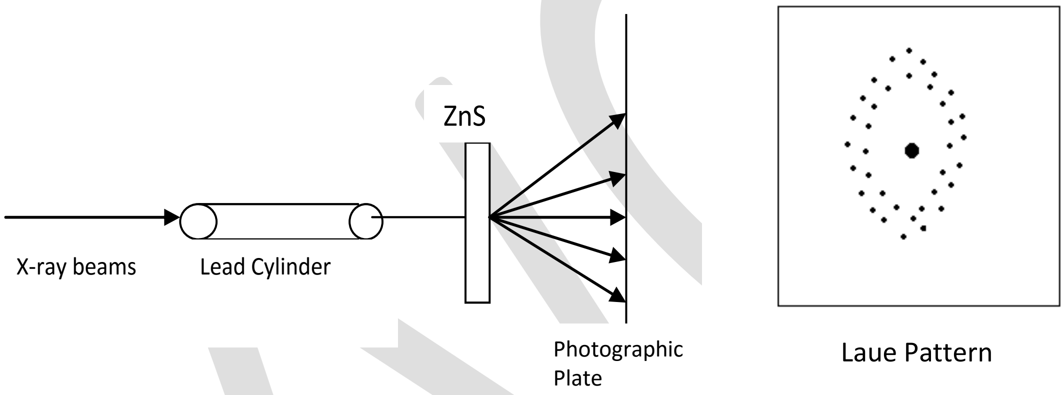
For constructive interference, the path difference must be an integral multiple of the wavelength.

$$\boldsymbol{2d\ \sin\theta = n\,\lambda}$$

This is known as the Bragg's law of X-ray Diffraction.

X-ray Diffraction by Laue Method

X-rays cannot be diffracted using a normal grating since their wavelength is very small. But Laue suggested that X-rays can be diffracted using a crystal. We know that the atoms in a crystal are arranged in a regular and periodic manner. Hence the crystal serves as a natural three dimensional grating.



The experimental arrangement for X-ray diffraction by Laue method is shown in figure. X-rays are passed through a thin lead cylinder to obtain a narrow beam of X-rays. The beam of X-rays falls on a stationery zinc sulphide crystal. The crystal acts as a natural three dimensional grating and diffracts the X-rays. The diffraction pattern is recorded on a photographic plate. The diffraction pattern contains a central black spot surrounded by a number of spots in a definite pattern. These spots are called Laue spots and the symmetrical pattern is known as Laue pattern.

X-rays used in this experiment have continuous range of wavelengths. Hence there will be many values of λ for which the Bragg condition is satisfied for any orientation (θ)of the lattice planes. The value of inter planar spacing 'd' is different for different sets of parallel planes. Hence diffraction pattern will be produced for each set of parallel planes.

Significance of the experiment:

1. This experiment proves that X-rays are electromagnetic waves having a certain wavelength. The wavelength of X-rays can be determined using this experiment.
2. Structure of the crystal can be studied by knowing the intensities and positions of the Laue spots.

Unit-V

SUPER CONDUCTIVITY

The electrical resistivity of some materials drops to zero when they are cooled below a certain temperature. This phenomenon is called Superconductivity and the materials are called super conductors. The temperature at which the resistivity becomes zero is called Critical temperature (T_c) 0r Transition temperature.

Examples: Mercury, Silver, Lead, Gallium, Iridium etc.

Basic experimental facts

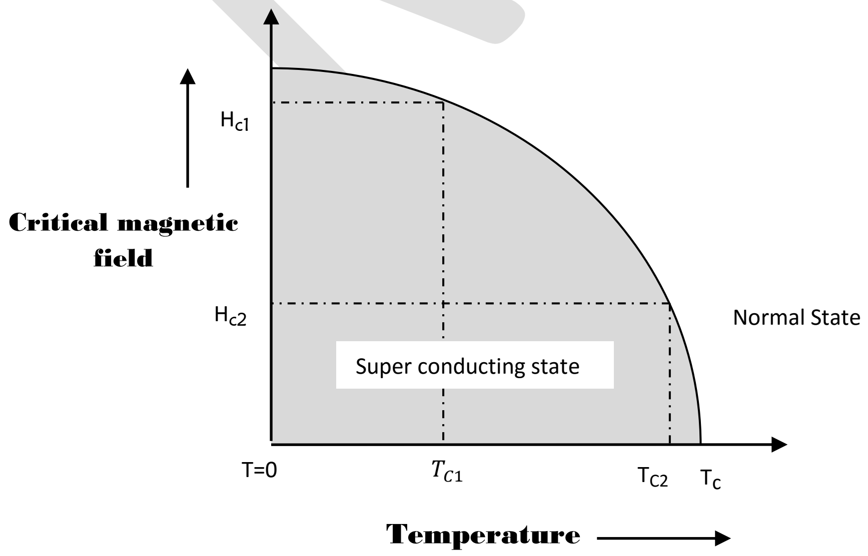
- The electrical resistivity of super conductors below critical temperature is zero.
- The resistivity of super conductors is high at normal temperatures.
- Super conductors are perfectly diamagnetic.
- The transition temperature is different for different isotopes of the same super conductors.
- Super conductivity of a material is not destroyed by the addition of impurities. But the critical temperature decreases.
- There is no change in the crystal structure when a material exhibits super conductivity. Hence super conductivity is due to the conduction electrons.
- When a weak magnetic field is applied on a super conductor, the critical temperature decreases.
- When a strong magnetic field is applied the material loses its super conductivity.

Effect of external Magnetic field

When a strong magnetic field is applied on a super conductor, it loses its super conductivity. The minimum value of magnetic field required to destroy the super conductivity of a material is called critical magnetic field (H_c).The variation of critical magnetic field with temperature is shown in figure. It can be represented by the following relation.

$$H_c(T) = H_c(0)\left[1-\frac{T^2}{T_c^2}\right]$$

H_c (T) represents the critical magnetic field at temperature T and H_c (0) is the critical field at absolute zero.



Isotope effect:

The transition temperature is different for different isotopes of the same super conductor. It decreases with increasing Isotopic mass.

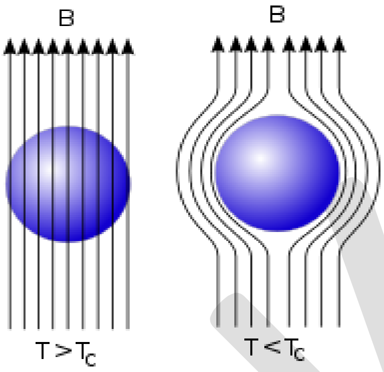
The transition temperature (T_c) is inversely proportional to the square root of isotopic mass (M).

$$T_c\, M^{1/2} = Constant$$

The transition temperature of mercury varies from 4.185k to 4.16k when the isotopic mass varies from 199.5 to 203.4 amu. The above equation tells us that the lattice vibrations are involved in super conductivity.

Meissner Effect

When a super conductor is cooled in presence of a magnetic field, the magnetic lines of force are expelled from the material below the critical temperature. This phenomenon is known as Meissner effect. Hence the magnetic induction inside a super conductor is zero below the critical temperature.



Meissner effect shows that super conductors exhibit perfect diamagnetism.

1. Meissner effect is reversible.
2. A super conductor is a perfect diamagnetic. The magnetic induction inside a super conductor is zero.

$$B = \mu_o(\,H + M\,) = 0$$

$$M = -H$$

Magnetic Susceptibility $\chi = \frac{M}{H} = \frac{-H}{H} = -1$

$$\chi = -1$$

3. Maxwell's Equation is given by

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial T}$$

From Ohm's law $V = I\,R$ and $E = \frac{V}{d}$

$$E = \frac{IR}{d} = \frac{JAR}{d} = J\left(\frac{AR}{d}\right)$$

$$E = J\rho$$

Where $\rho = \frac{AR}{d}$

Since ρ is zero for a super conductor, $E = o$

$$\frac{\partial B}{\partial T} = 0$$

$$\boldsymbol{B = Constant}$$

Hence the magnetic field inside a perfect conductor is not zero. This contradicts Meissner effect. Hence it is clear that perfect diamagnetism of a super conductor is not due to its perfect conducting property.

Hence the two independent properties defining a super conductor are

1. Perfect Conductor
2. Perfect Diamagnetic.

Type I and Type II Super Conductors

Super conductors are classified into two types basing on their magnetic behaviour. They are Type I and Type II Super conductors.

Type I Super conductors

The variation of intensity of magnetization (I) of a type I super conductor with applied magnetic field is shown in figure. It is clear from the figure that the magnetization of the super conductor increases with H. The magnetization quickly drops to zero when the magnetic field exceeds the critical value H_C and becomes normal.

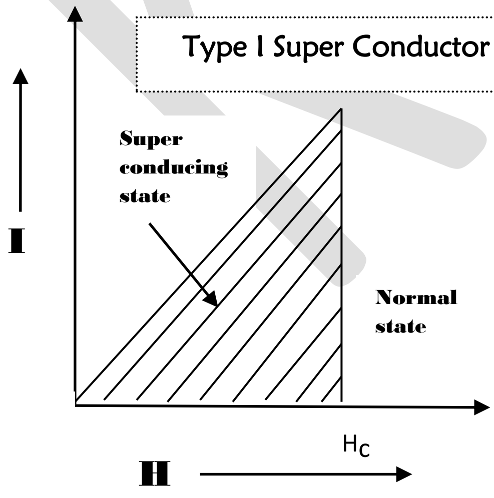
In type I super conductors, the transition from super conducting to normal state takes place sharply at H_C. The material is in super conducting state when H is less than H_C and in normal state when H exceeds H_C. Hence Type I superconductors strictly obey Meissner effect.

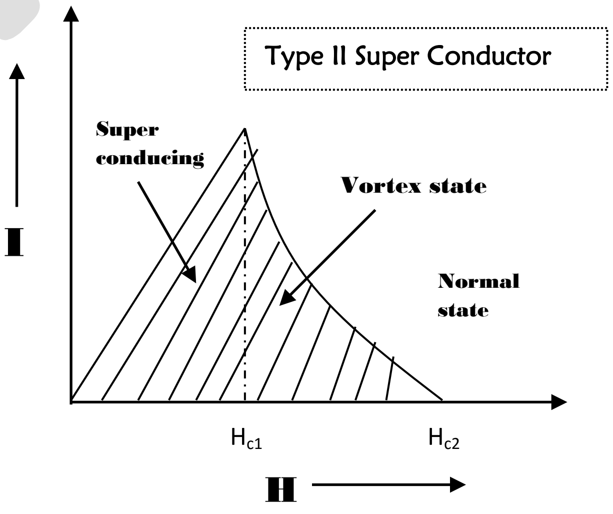
Type II Super conductors

The variation of intensity of magnetization of a type II super conductor with applied magnetic field is shown in figure. It is clear from the figure that type II super conductor has two critical magnetic fields H_{C1}, H_{C2}

1. When the magnetic field is less than H_{C1}, the super conductor behaves as a perfect dia magnet. H_{c1} is called lower critical field.
2. When the magnetic field exceeds H_{C1}, the magnetic lines of force begin to penetrate the material. The magnetization becomes completely zero when the magnetic field reaches H_{C2}. Hence between H_{C1} and H_{C2} the material is in a mixed structure of normal and superconducting states. The mixed state is called vortex state. In this state the material is a perfect conductor but not a perfect diamagnet.
3. After H_{C2}, the material changes to normal state.

Hence type II superconductors do not strictly obey Meissner effect and have two critical magnetic fields H_{C1}, H_{C2}.





H_C